Space Shift Keying (SSK) Modulation: On the Transmit–Diversity / Multiplexing Trade–Off

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***Abstract*— Space Shift Keying (SSK) is a low–complexity modulation scheme for multiple–antenna wireless systems. In this paper, we analyze the transmit–diversity/multiplexing trade–off of SSK modulation with the main objective of developing practical solutions to achieve transmit–diversity. More specifically, the contributions of this paper are as follows: i) we propose a practical scheme that achieves transmit–diversity equal to two for any number of antennas at the transmitter. The solution is based on the so–called Time–Orthogonal–Signal–Design (TOSD) principle introduced in [1], and adopts time–orthogonal shaping filters at the transmitter; ii) we show that the TOSD principle with orthogonal shaping filters can be applied to the so–called Generalized SSK (GSSK) modulation scheme in [2], and that a transmit–diversity equal to two can still be obtained while increasing the data rate with respect to SSK modulation; and iii) we propose a general encoding scheme that allows us to get transmit–diversity greater than two. The solution combines TOSD and GSSK principles in a unique fashion, and is flexible enough to accommodate various transmit–diversity gains by trading–off the number of transmit–antenna, the number of simultaneously–active transmit–antenna, and the achievable data rate. Furthermore, proposed methods and findings are substantiated via analysis and numerical simulations.**

I. Introduction

Spatial Modulation (SM) is a recently proposed modulation scheme for multiple–antenna wireless systems, which increases the data rate of single–antenna systems (multiplexing gain) without the need of multiplexing multiple data–streams at the transmitter and, thus, avoiding multi–stream detectors at the receiver [3]–[6]. The fundamental benefit introduced by SM for multiple–antenna wireless systems can be readily understood by regarding this technology as a high–rate coding mechanism [7, Eq. (1)]. SM uses the spatial domain as an additional dimension (the so–called spatial–constellation diagram), on top of the conventional signal–constellation diagram [5], to convey part of the information bits. This is realized by exploiting the spatial de–correlation property of the wireless medium for data modulation, which allows the encoder to establish a one–to–one mapping between the information messages and the channel impulse responses on the available transmit–to–receive wireless links [6].

However, it is well–known that the performance of a given transmission technology is only in part determined by the multiplexing gain, and that another important component to be analyzed is the so–called diversity gain [8]. Furthermore, a large multiplexing gain can be easily offset by a small diversity gain. Thus, it is very important to study the diversity offered by SM and to understand the multiplexing/diversity trade–off provided by this technology. This problem has recently attracted the interest of some researchers. More specifically, most research is focused on a low complexity implementation of SM, which is known as Space Shift Keying (SSK) modulation [9]. Unlike SM, in SSK modulation only the spatial–constellation diagram is used for data modulation, thus trading–off transmitter and receiver complexity for the achievable data rate [5]. In [9] and [10], it is shown that SSK modulation can achieve a receive–diversity gain that increases linearly with the number of antennas at the receiver. In [2], it is shown that, regardless of the number of simultaneously–active antennas at the transmitter, SSK modulation is unable to provide transmit–diversity gains. In [1], a simple method is introduced to overcome that limitation. The solution is applicable to a transceiver with two transmit–antenna and one receive–antenna, and it neither results in a loss of spectral efficiency, nor does it require multiple simultaneously–active antennas at the transmitter. In [11], transmit–diversity is achieved by sending redundant information in non–overlapping time–slots, and thus resulting in a spectral efficiency loss. In [12], it is proved that the method in [1] is unable to provide full–diversity for an arbitrary number of antennas at the transmitter and, in general, it allows us to achieve transmit–diversity only equal to two. Finally, in [13] the authors have studied the achievable transmit–diversity of SM and have pointed out that SM cannot achieve transmit–diversity. However, no solutions are provided to cope with this issue and it is shown that the absence of transmit–diversity may result, especially for high correlated fading channels, in a substantial performance loss. The design of transmit–diversity for SM is investigated in [14], and a simple solution to achieve transmit–diversity equal to two is proposed. Another solution can be found in [15].

From all the above, it is apparent that while receive–diversity is well understood for SM and SSK modulation, transmit–diversity is an open research issue, which deserves further investigation for the successful application of this technology especially in the downlink of wireless communication systems. In fact, in this scenario it is more economical to add complexity to a single central entity rather than at multiple remote and primarily low–cost devices. Motivated by these considerations, this paper aims at shedding light on the design of transmit–diversity for SSK modulation. The specific contributions of this paper are as follows. 1) We move from the Time–Orthogonal–Signal–Design (TOSD) principle in [1], and propose a practical method to design a SSK modulation scheme with transmit–diversity equal to two for any number of antennas at the transmitter. The method uses time–orthogonal waveforms to shape the signals emitted by the antenna–array at the transmitter. 2) In order to increase the data rate of SSK modulation without reducing the performance too much, we show that the method in 1) can be applied to Generalized SSK (GSSK) modulation [2], which allows multiple transmit–antenna to be simultaneously active for data transmission. This additional degree of freedom comes at the expenses of increasing both transmitter and receiver complexity. However, single–stream detection can still be used at the receiver. 3) Finally, we propose an advanced transmit–diversity scheme which combines TOSD and GSSK modulation in a unique fashion, and allows us to achieve transmit–diversity gains greater than two by adequately choosing the number of transmit–antenna and active transmit–antenna. The price to be paid for this flexibility is twofold [6]: i) multiple antennas at the transmitter have to transmit data at the same time, and ii) a reduction in the achievable data rate with respect to the maximum rate achieved by GSSK modulation. This results in a transmit–diversity/multiplexing trade–off that is accurately investigated in this paper. Furthermore, we emphasize that our solutions still retain a single–stream receiver for data detection regardless of the number of simultaneously–active antennas at the transmitter. Finally, we note that the results described in this paper are novel in different ways: i) with respect to [1], we provide a practical scheme to achieve transmit–diversity and do not limit ourselves to merely identifying the general conditions that the transmitted pulses should satisfy for performance improvement, ii) for the first time, we report a transmit–diversity method for GSSK modulation, and iii) we document, for the first time for SSK modulation, a coding scheme with transmit–diversity greater than two, which does not exploit spectrally inefficient repetition coding.

The reminder of this paper is organized as follows. In Section II, we review SSK and GSSK modulation schemes in order to highlight their limits in achieving transmit–diversity. In Section III, we propose a general method to design SSK and GSSK modulation schemes with transmit–diversity equal to two. In Section IV, we extend the analysis in Section III to design SSK modulation schemes with transmit–diversity greater than two. In Section V, we provide some general guidelines to designing SSK modulation schemes with arbitrary transmit–diversity and study the related transmit–diversity/multiplexing trade–off. In Section VI, we analyze differences and similarities of our proposed transmit–diversity schemes with respect to conventional methods. In Section VII, our claims are substantiated through Monte Carlo simulations. Finally, Section VIII concludes the paper.

II. Background: SSK Modulation with Transmit–Diversity 1

A. System Model

We consider a general Multiple–Input–Single–Output (MISO) communication system with N_t antennas at the transmitter and $N_r = 1$ antennas at the receiver. The assumption $N_r = 1$ does not limit the generality of the results derived in this paper since we are mainly interested in studying transmit–diversity. From [10], it can be readily proved that the solutions described in this paper can be extended to multiple receive–antenna and that the overall diversity achieved by the resulting system is simply multiplied by N_r. We assume that the receiver uses a Maximum–Likelihood (ML) detector with Full Channel State Information (F–CSI) [16].

Notation. The following notation is used throughout this paper: i) $|\cdot|^2$ and $\|\cdot\|^2$ denote the square absolute value of a complex number and the square Euclidean norm of a complex vector, respectively; ii) N_a is the number of simultaneously–active antennas at the transmitter, with $1 \leq N_a \leq N_t$; iii) E_m is the average total energy transmitted by the N_a active antennas that emit a non–zero signal. E_m is equally distributed among the active antennas, *i.e.* each active antenna emits a signal with energy E_m/N_a; iv) N_0 is the power spectral density per dimension of the Additive White Gaussian Noise (AWGN) at the receiver input; v) $\bar{\gamma}{=}E_m/(4N_0)$; vi) $Q(x) = \left(1/\sqrt{2\pi}\right)\int_x^{+\infty} \exp\left(-t^2/2\right) dt$ is the Q–function; vii) $\{\alpha_i\}_{i=1}^{N_t}$ is the complex channel gain on the wireless link from the i–th transmit–antenna to the receive–antenna; viii) $\{w_i(\cdot)\}_{i=1}^{N_t}$ is the pulse shape used at the i–th transmit–antenna. Unless otherwise stated, we assume: $\int_{-\infty}^{+\infty} w_i(t)\, w_j(t)\, dt = 0$ if $i \neq j$ and $\int_{-\infty}^{+\infty} w_i(t)\, w_j(t)\, dt = 1$ if $i = j$; ix) R denotes the rate in bits/s/Hz; x) $\lfloor\cdot\rfloor$ is the floor function; and xi) $\binom{\cdot}{\cdot}$ is the binomial coefficient.

B. SSK Modulation

SSK modulation works as follows [6], [9]: i) the transmitter encodes blocks of $\log_2(N_t)$ data bits into the index of a single transmit–antenna (*i.e.*, $N_a = 1$), which is switched on for data transmission while all the other antennas are kept silent, and ii) the receiver solves a N_t–hypothesis testing problem to estimate the transmit–antenna that is not idle, which results in the estimation of the unique sequence of bits emitted by each encoder [10, Sec. III]. The block of bits encoded into the index of the i–th transmit–antenna is called "message", and the N_t messages are equiprobable. Furthermore, in SSK modulation the shaping filters used by the transmit–antenna are the same, *i.e.*, $w_i(t) = w_0(t)$ for $i = 1, 2, \ldots, N_t$, and $\int_{-\infty}^{+\infty} w_0(t)\, w_0(t)\, dt = 1$.

In [6], we have shown that the Bit Error Probability (BEP) of SSK modulation can be tightly upper–bounded as follow:

$$\mathrm{BEP_{SSK}} \leq \frac{1}{N_t - 1}\sum_{t_1=1}^{N_t}\sum_{t_2=t_1+1}^{N_t} Q\left(\sqrt{\bar{\gamma}\,|\alpha_{t_2} - \alpha_{t_1}|^2}\right) \quad (1)$$

We emphasize that (1) is conditioned upon fading channel statistics. The Average Bit Error Probability (ABEP) can be computed either numerically or analytically [6], [10].

From (1), we conclude that SSK modulation has transmit–diversity equal to one. In fact, each term in the two–fold summation depends on the difference of two complex channel gains, which turns out to be equivalent to a Single–Input–Single–Output (SISO) system with an equivalent channel gain given by the difference of them. From [8], it can be concluded that SSK achieves no transmit–diversity gain.

C. GSSK Modulation

In [2], the authors have introduced GSSK modulation, which is a generalized version of SSK modulation that does not restrict the number of simultaneously–active antennas to $N_a = 1$. With respect to [2], in this paper we have a different view of the usefulness of GSSK modulation for multiple–antenna wireless systems. In [2], the GSSK concept is proposed as a modulation scheme that exploits CSI at the transmitter for optimizing the spatial–constellation diagram. Numerical results have shown some performance improvements with respect to SM, but at the cost of requiring a feedback channel. In this paper, we use GSSK modulation *without* CSI at the transmitter. In our opinion, the main flexibility introduced by the GSSK concept is not in the optimization of the spatial–constellation diagram for performance improvement, but in the inherent possibility of switching on multiple antennas at the transmitter for increasing the data rate. In fact, this flexibility, which comes at the expenses of transmitter complexity [9], allows GSSK modulation to enlarge the size of the spatial–constellation diagram, and, thus, to increase the achievable data rate. In particular, while the achievable rate of SSK modulation is $R_{\rm SSK} = \log_2(N_t)$, GSSK modulation can provide a data rate up to $R_{\rm GSSK} = \left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor$, where the floor function stems form the fact that the constellation size is constrained to be a power of two to use the SSK principle. It can be shown that $R_{\rm GSSK}$ achieves its maximum value when $N_a = \lfloor N_t/2 \rfloor$. So, in this paper we are mainly interested in studying transmit–diversity schemes for GSSK modulation that do not require any CSI at the transmitter.

The working principle of GSSK modulation used in this paper can be summarized as follows: i) the transmitter encodes blocks of $\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor$ bits into one point of an enlarged spatial–constellation diagram of size $N_H = 2^{\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor}$, which enables N_a antennas to be switched on for data transmission while all the other antennas are kept silent, and ii) similar to SSK modulation, the receiver solves a N_H–hypothesis testing problem to estimate the N_a antennas that are not idle, which results in the estimation of the unique message emitted by the encoder. Similar to SSK modulation, $w_i(t) = w_0(t)$ for $i = 1, 2, \ldots, N_t$, and $\int_{-\infty}^{+\infty} w_0(t) w_0(t)\, dt = 1$.

Under these assumptions, the performance of GSSK modulation can be estimated by using the result summarized in *Theorem 1*. We note that the BEP in (2) is much tighter than the framework provided in [2] for the same reasons as those provided in [6] for SSK modulation.

Theorem 1: Let $S_{\rm comb}$ be the set of N_a–combination of the set of N_t antennas at the transmitter. The size of $S_{\rm comb}$ is $\binom{N_t}{N_a}$. Let $\bar{\alpha}_k$ denote the k–th element of $S_{\rm comb}$ for $k = 1, 2, \ldots, \binom{N_t}{N_a}$. $\bar{\alpha}_k$ is a N_a–dimension vector whose elements ($\bar{\alpha}_k(q)$ for $q = 1, 2, \ldots, N_a$) are the fading coefficients $\{\alpha_i\}_{i=1}^{N_t}$. Then, the BEP can be upper–bounded as follows:

$$\mathrm{BEP_{GSSK}} \le \frac{1}{N_H - 1} \sum_{t_1=1}^{N_H} \sum_{t_2=t_1+1}^{N_H} Q\left(\sqrt{\frac{\bar{\gamma}}{N_a} \left| \sum_{q=1}^{N_a} \left[\bar{\alpha}_{t_2}(q) - \bar{\alpha}_{t_1}(q) \right] \right|^2} \right) \tag{2}$$

where we have assumed that the first N_H (with $N_H < \binom{N_t}{N_a}$) elements of $S_{\rm comb}$ have been chosen to implement the GSSK modulation scheme (no optimization on the spatial–constellation diagram is considered).

Proof: The result in (2) follows from the analytical development in [6], [10], and by taking into account that: i) the equivalent transmitted message is given by the summation of the signals emitted by the N_a active transmit–antenna, ii) the spatial–constellation diagram has size N_H, and iii) the energy emitted by each antenna is scaled by N_a to keep constant the total radiated energy per transmission. □

To clarify the notation in *Theorem 1*, let us consider a simple example with $(N_t, N_a) = (5, 2)$. In this case, we have: $\bar{\alpha}_1 = [\alpha_1, \alpha_2]$, $\bar{\alpha}_2 = [\alpha_1, \alpha_3]$, $\bar{\alpha}_3 = [\alpha_1, \alpha_4]$, $\bar{\alpha}_4 = [\alpha_1, \alpha_5]$, $\bar{\alpha}_5 = [\alpha_2, \alpha_3]$, $\bar{\alpha}_6 = [\alpha_2, \alpha_4]$, $\bar{\alpha}_7 = [\alpha_2, \alpha_5]$, $\bar{\alpha}_8 = [\alpha_3, \alpha_4]$, $\bar{\alpha}_9 = [\alpha_3, \alpha_5]$, $\bar{\alpha}_{10} = [\alpha_4, \alpha_5]$. In (2), only the first $N_H = 8$ elements of $S_{\rm comb}$, *i.e.*, $\bar{\alpha}_k$ for $k = 1, 2, \ldots, N_H$ are considered. Also, the ABEP can be computed either from (2) by using the framework developed in [10] or numerically.

Finally, similar to SSK modulation, from (2) it is simple to conclude that no transmit–diversity is achieved by GSSK modulation. In fact, each term in the two–fold summation in (2) depends on the linear combination of $2N_a$ channel gains, which turns out to be equivalent to a SISO system with a channel gain equal to this linear combination. Thus, with respect to SSK modulation, GSSK modulation can increase the data rate, but it is still unable to provide transmit–diversity.

III. SSK Modulation with Transmit–Diversity 2

The aim of this section is to propose improved schemes that can overcome the limitations of SSK and GSSK modulation to achieve transmit–diversity. More specifically, we propose a general method to achieve transmit–diversity equal to two for arbitrary values of N_t and N_a. The method is based on the TOSD principle introduced in [1], where we have shown that transmit–diversity can be achieved via adequate pulse shaping at the transmitter. However, we significantly improve the intuition in [1]. More precisely, in [1] the orthogonal signal design exploits the different propagation delays on the transmit–to–receive wireless links. However, this solution could require a signal with a very large transmission bandwidth. In this paper, we propose a different way to exploit the TOSD principle, which foresees time–orthogonal shaping filters across the transmit–antenna. Furthermore, we show that this approach can be used for any N_t, while in [1] only the setup with $N_t = 2$ is considered. Finally, we show that the proposed idea can be extended to GSSK modulation.

A. TOSD–SSK Modulation

By using a terminology similar to [1], the proposed modulation scheme is called TOSD–SSK modulation. Its working principle is the same as SSK modulation in Section II-B, but with a fundamental difference: each antenna, when active for data transmission, radiates a different pulse waveform, and the waveforms across the antennas are time–orthogonal as described in Section II-A. We emphasize that in TOSD–SSK modulation a single antenna is active for data transmission, and that the transmitted message is still encoded into the index of the transmit–antenna and not into the impulse response of the shaping filter. In other words, the proposed idea is different from conventional SISO schemes, which use Orthogonal Pulse Shape Modulation (O–PSM) [17] and are unable to achieve transmit–diversity, as only a single wireless link is exploited for communication.

The performance of TOSD–SSK modulation can be estimated by using the result summarized in *Theorem 2*.

Theorem 2: The BEP of TOSD–SSK modulation with time–orthogonal shaping filters is upper–bounded as follows:

$$\mathrm{BEP_{TOSD-SSK}} \le \frac{1}{N_t - 1} \sum_{t_1=1}^{N_t} \sum_{t_2=t_1+1}^{N_t} Q\left(\sqrt{\bar{\gamma} \left(|\alpha_{t_1}|^2 + |\alpha_{t_2}|^2 \right)} \right) \tag{3}$$

Proof: The result in (3) follows from [10] by taking into account that: i) by exploiting the orthogonality of the shaping filters, the cross–product of the complex channel gains in [10, Eq. (9)] is always equal to zero, and ii) the noises at the output of the matched filters in [10, Eq. (11)] are uncorrelated due to the orthogonality of the shaping filters. □

By carefully analyzing (3), we observe that, unlike SSK modulation, each summand depends on the power–sum of two channel gains. According to [8], it can be shown that this system offers a transmit–diversity equal to two. We notice that if $N_t = 2$ the system achieve full transmit–diversity. The ABEP can be computed by using well–consolidated frameworks for performance analysis of receive–diversity systems [8].

B. TOSD–GSSK Modulation

Let us now generalize the GSSK modulation scheme for transmit–diversity. The new modulation concept is called TOSD–GSSK modulation, and exploits the same modulation principle as GSSK in Section II-C but uses time–orthogonal shaping filters at the transmitter. The performance of TOSD–GSSK modulation can be estimated from *Theorem 3*.

Theorem 3: By adopting the same notation as in *Theorem 1*, the BEP of TOSD–GSSK modulation with time–orthogonal shaping filters is upper–bounded as follows:

$$\mathrm{BEP}_{\mathrm{TOSD-GSSK}} \le \frac{1}{N_H - 1} \sum_{t_1=1}^{N_H} \sum_{t_2=t_1+1}^{N_H} Q\left(\sqrt{\frac{\bar{\gamma}}{N_a} \left\| \bar{\alpha}_{t_1,t_2} \right\|^2}\right) \quad (4)$$

where $\bar{\alpha}_{t_1,t_2}$ is a vector whose components are all the distinct elements in $\bar{\alpha}_{t_1}$ and $\bar{\alpha}_{t_2}$.

Proof: The result in (4) follows immediately by taking into account the comments in *Theorem 1* and *Theorem 2*. Due to space constraints, the details of the derivation are omitted. □

To clarify the notation in (4) and to better understand the achievable performance, we consider again the example in Section II-C with $(N_t, N_a) = (5, 2)$. We have, *e.g.*: $\bar{\alpha}_{1,2} = [\alpha_2, \alpha_3]$, $\bar{\alpha}_{1,3} = [\alpha_2, \alpha_4]$, $\bar{\alpha}_{1,8} = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]$, *etc.* By carefully analyzing (4) and the example, we observe that, unlike GSSK modulation, each term in (4) depends on the power–sum of *at least* two channel gains. Similar to TOSD–SSK modulation, we can conclude that the system can provide a transmit–diversity equal to two. Two important observations are worth being made in this case: i) the fact that the power–sum of *at least* two channel gains is obtained is inherent in the "data bit" to "N_a–active transmit–antenna index" mapping of GSSK modulation. In other words, for each pair of transmitted messages, the two subsets of active antennas contain at least two different transmit–antenna indexes; and ii) some terms in (4) have higher transmit–diversity. For example, the term corresponding to $\bar{\alpha}_{1,8}$ in the example above has transmit–diversity equal to four. However, it is known that the overall transmit–diversity of the system depends on the worst summands in (2), *i.e.*, the terms with transmit–diversity two. Similar to TOSD–SSK modulation, the ABEP can be easily computed [8].

IV. SSK Modulation with Transmit–Diversity Greater than 2

The main drawback of the methods introduced in Section III is their limitation to provide transmit–diversity greater than two, even when $N_t > 2$ and $N_a > 2$. Basically, in TOSD–SSK and TOSD–GSSK modulation, N_t and N_a can only be used to adjust the data rate, which is equal to $R_{\mathrm{TOSD-SSK}} = \log_2(N_t)$ and $R_{\mathrm{TOSD-GSSK}} = \left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor$, respectively. In this section, we describe a method that combines TOSD and GSSK principles to achieve a higher transmit–diversity. The main idea is based on the TOSD–GSSK modulation scheme, but instead of allowing the transmitter to exploit the *whole* spatial–constellation diagram (*i.e.*, the N_H message–to–antenna mappings), we adequately choose a subset of points, $N_H^{\perp} < N_H$, where to apply the SSK modulation principle. This limits the achievable data rate, which reduces to $R = \log_2(N_H^{\perp})$, but allows us to increase the transmit–diversity. For reasons that will become apparent in the next sub–sections, the proposed method is called: *TOSD–GSSK modulation with mapping by pairwise disjoint set partitioning*. We emphasize that the choice of the spatial–constellation diagram is different from [2], which is unable to provide transmit–diversity, and requires no CSI at the transmitter.

In this section, we introduce our idea with two examples. The general procedure is described in Section V along with the analysis of the transmit–diversity/multiplexing trade-off.

A. Transmit–Diversity 4 – An Example

Let us describe a simple scheme with $N_t = 4$, $N_a = 2$, and $R = 1$, which provides transmit–diversity equal to four. Similar to the TOSD–GSSK modulation principle, we assume that the N_t antennas use time–orthogonal shaping filters.

The working principle is as follows[1]: i) if the encoder emits a "0" ("1") bit, the SSK mapper encodes it into the pair of antennas $\{\mathrm{TX}_1, \mathrm{TX}_2\}$ ($\{\mathrm{TX}_3, \mathrm{TX}_4\}$), which are switched on for transmission, while the antennas $\{\mathrm{TX}_3, \mathrm{TX}_4\}$ ($\{\mathrm{TX}_1, \mathrm{TX}_2\}$) are kept silent; and ii) the receiver uses a single–stream ML–optimum detector similar to TOSD–GSSK modulation.

The BEP can be computed by using a methodology similar to *Theorem 3*. The final result is given in *Corollary 1*.

Corollary 1: The BEP of the $(N_t, N_a, R) = (4, 2, 1)$ scheme with TOSD–GSSK modulation with mapping by pairwise disjoint set partitioning is as follows:

$$\mathrm{BEP}_{\mathrm{Div4}} = Q\left(\sqrt{\frac{\bar{\gamma}}{N_a} \sum_{t=1}^{N_t} |\alpha_t|^2}\right) \quad (5)$$

The formula in (5) confirms that the transmit–diversity achieved by the proposed scheme is four, *i.e.*, full transmit–diversity is obtained in this case.

B. Transmit–Diversity 6 – An Example

Let us describe a simple scheme with $N_t = 12$, $N_a = 3$, and $R = 2$, which provides transmit–diversity equal to six. Also in this case, we assume that the N_t antennas use time–orthogonal shaping filters.

The working principle is as follows: i) If the encoder emits a "00" ("01", "10", "11") pair of bits, the SSK mapper encodes them into the triple of antennas $\{\mathrm{TX}_1, \mathrm{TX}_2, \mathrm{TX}_3\}$ ($\{\mathrm{TX}_4, \mathrm{TX}_5, \mathrm{TX}_6\}$, $\{\mathrm{TX}_7, \mathrm{TX}_8, \mathrm{TX}_9\}$, $\{\mathrm{TX}_{10}, \mathrm{TX}_{11}, \mathrm{TX}_{12}\}$), which are switched on for transmission, while all the other antennas are kept silent; and ii) the receiver uses a single–stream ML–optimum detector similar to TOSD–GSSK modulation.

The BEP can be computed by still using *Theorem 3*. The final result is given in *Corollary 2*.

Corollary 2: The BEP of the $(N_t, N_a, R) = (12, 3, 2)$ scheme with TOSD–GSSK modulation with mapping by pairwise disjoint set partitioning is upper–bounded as follows:

$$\mathrm{BEP}_{\mathrm{Div6}} \le \frac{1}{N_H^{\perp} - 1} \sum_{t_1=1}^{N_H^{\perp}} \sum_{t_2=t_1+1}^{N_H^{\perp}} Q\left(\sqrt{\frac{\bar{\gamma}}{N_a} \mathrm{SNR}_{t_1,t_2}}\right) \quad (6)$$

where $N_H^{\perp} = 4$, and $\mathrm{SNR}_{1,2} = |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 + |\alpha_5|^2 + |\alpha_6|^2$, $\mathrm{SNR}_{1,3} = |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_7|^2 + |\alpha_8|^2 + |\alpha_9|^2$, $\mathrm{SNR}_{1,4} = |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 + |\alpha_{12}|^2$, $\mathrm{SNR}_{2,3} = |\alpha_4|^2 + |\alpha_5|^2 + |\alpha_6|^2 + |\alpha_7|^2 + |\alpha_8|^2 + |\alpha_9|^2$, $\mathrm{SNR}_{2,4} = |\alpha_4|^2 + |\alpha_5|^2 + |\alpha_6|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 + |\alpha_{12}|^2$, $\mathrm{SNR}_{3,4} = |\alpha_7|^2 + |\alpha_8|^2 + |\alpha_9|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 + |\alpha_{12}|^2$.

From (6), we conclude that the proposed method achieves a very high transmit–diversity equal to six. Furthermore, this

[1]We denote by TX_i $(i = 1, 2, \ldots, N_t)$ the N_t antennas at the transmitter.

transmit–diversity gain is achieved with only three active antennas at the transmitter.

V. Transmit–Diversity / Multiplexing Trade–Off

Let us now summarize in two general theorems the transmit–diversity methods illustrated through examples in Section III and Section IV. These theorems provide a general procedure to design SSK modulation schemes with the desired transmit–diversity/multiplexing trade–off.

Theorem 4: Let a (N_t, N_a) multiple–antenna wireless system using SSK modulation. Let $N_H = 2^{\left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor}$ be the size of the spatial–constellation diagram. Then: i) the system achieves transmit–diversity equal to $\mathrm{Div} = 1$ and rate $R = \left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor$ if the N_t antennas at the transmitter use the same shaping filter. This scheme is called GSSK modulation and reduces to SSK modulation if $N_a = 1$; and ii) the system achieves transmit–diversity equal to $\mathrm{Div} = 2$ and rate $R = \left\lfloor \log_2 \binom{N_t}{N_a} \right\rfloor$ if the N_t transmit–antenna use time–orthogonal shaping filters. This scheme is called TOSD–GSSK modulation and reduces to TOSD–SSK modulation if $N_a = 1$.

Proof: The proof follows immediately from Section III. □

Theorem 5: Let a (N_t, N_a) multiple–antenna wireless system using the SSK modulation principle. Let $N_H^{\perp}$ be the size of the partition[2] of the set of N_t antennas such that $N_t = N_H^{\perp} N_a$, *i.e.*, each subset of the partition has N_a distinct elements and the subsets are pairwise disjoint. Then, the system achieves transmit–diversity equal to $\mathrm{Div} = 2N_a$ and rate $R = \log_2 \left(N_H^{\perp} \right)$ if the N_t transmit–antenna use time–orthogonal shaping filters. This scheme is called TOSD–GSSK modulation with mapping by pairwise disjoint set partitioning.

Proof: The proof follows immediately from Section IV. □

VI. Differences and Similarities with Conventional Transmit–Diversity

In this section, we aim at providing some insights about differences and similarities among the proposed transmit–diversity schemes and those available in the literature for conventional modulation. By looking at some state–of–the–art proposals for transmit–diversity [18], we can readily recognize that the most similar solution to the transmit–diversity concepts for SSK modulation proposed in Section III and Section IV is the so–called Orthogonal Transmit–Diversity (OTD) method [18, Fig. 3]. In OTD, the signals emitted by multiple antennas at the transmitter are shaped by using time–orthogonal shaping filters[3], as the TOSD principle in our proposals foresees. However, TOSD for SSK modulation and OTD are very different in the way transmit–diversity is achieved. The reason is threefold. 1) Let us consider the TOSD–SSK modulation scheme in Section III-A. In *Theorem 4*, it is shown that this scheme can achieve $\mathrm{Div} = 2$ with a single active transmit–antenna, *i.e.*, $N_a = 1$. This is a property that is not shared with OTD, which achieves a transmit–diversity equal to the number of active antennas. In other words, TOSD–SSK modulation can achieve transmit–diversity even though there is only one active antenna, and this peculiar property stems from the SSK modulation principle, *i.e.*, in conveying the information bits into the spatial positions of the transmit–antenna. 2) The difference between our proposal and the OTD scheme can be understood even better by considering the TOSD–GSSK modulation with mapping by pairwise disjoint set partitioning in Section IV. In *Theorem 5*, it is shown that the transmit–diversity provided by this scheme is $\mathrm{Div} = 2N_a$, which means that the transmit–diversity is twice the number of active transmit–antenna. This is in net contrast to the OTD concept where the achievable diversity is equal to N_a. Again, this is due to the antenna–index coded modulation principle inherent in SSK modulation. This result tells us that with the proposed approach we can achieve the same transmit–diversity as the OTD scheme but we can halve the number of active antenna elements, and, thus, the number of radio frequency chains at the transmitter, which is known to be a very desirable feature to reduce the complexity and the power consumption of the transmitter [9]. 3) *Theorem 4* states that for SSK modulation is not sufficient to use time–orthogonal shaping filters at the transmitter to get transmit–diversity of any order, but the way the mapping "information–bit" to "spatial–constellation–point" is performed plays a crucial role to this end. More specifically, TOSD–GSSK modulation requires time–orthogonal filters in all the antennas at the transmitter, however it is unable to achieve transmit–diversity greater than two if the mapping by pairwise disjoint set partitioning is not used. In OTD, orthogonal pulse shaping is sufficient to achieve full transmit–diversity.

In conclusion, SSK modulation shares some features with OTD. However, it has some additional degrees of freedom to achieve transmit–diversity, which can be exploited by paying attention to the way the SSK modulator maps messages to points in the spatial–constellation diagram.

VII. Numerical Examples

In this section, numerical examples are shown to validate the claims in the sections above. The setup is as follows: i) we consider a frequency–flat Rayleigh channel model with independent and identically distributed fading over all the transmit–to–receive wireless links; ii) we assume the mean power of each fading gain to be normalized to 1; iii) to have a large set of orthogonal shaping filters, we use the family of Hermite polynomials [17]; and iv) as a performance metric, we consider the ABEP, which is obtained by averaging the BEP discussed in the sections above over fading channel statistics.

In Fig. 1, we compare Monte Carlo simulations and the analytical models developed in this paper[4]. We can observe a very good agreement between analysis and simulation, especially in the high E_m/N_0 region where our bounds are tighter. The slopes of the curves confirm our findings about the transmit–diversity gain achieved by the proposed schemes. In particular, we can notice that the slope of the ABEP gets steeper for increasing values of the transmit–diversity, which agrees with our analytical findings.

In Fig. 2, we analyze the performance of various proposed schemes for the same rate R. Numerical results confirm that the proposed schemes (*Theorem 5*) with transmit–diversity six and eight provide a substantial performance improvement with respect to already reported SSK modulation schemes with transmit–diversity one and two [1]. The price to pay is the need of increasing N_t and N_a. However, this characteristic is shared with other state–of–the–art transmit–diversity schemes, such as OTD and Space Time Block Codes (STBCs) [18].

[2]A partition of a set X is a set P of non–empty subsets of X such that the union of the elements of P is equal to X, and the intersection of any two distinct subsets of P is empty.

[3]In [18] the time–orthogonal shaping filters are Walsh codes typically used in spread spectrum systems.

[4]The framework for Div=8 is not shown here due to space constraints.

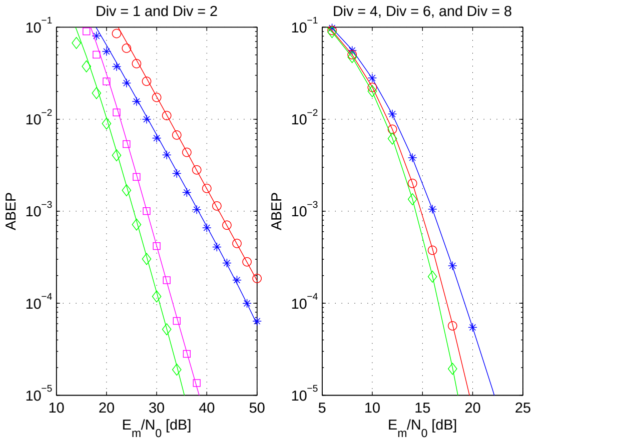

Fig. 1. ABEP against E_m/N_0: comparison of analytical model and simulation. Solid lines show the analytical model and markers show Monte Carlo simulations. (left) Diversity 1 and 2; Legend: ($*$) Div=1, $N_t = 5$, $N_a = 2$, $R = 3$ (GSSK) ($\circ$) Div=1, $N_t = 6$, $N_a = 3$, $R = 4$ (GSSK) ($\diamond$) Div=2, $N_t = 5$, $N_a = 2$, $R = 3$ (TOSD–GSSK) ($\square$) Div=2, $N_t = 6$, $N_a = 3$, $R = 4$ (TOSD–GSSK). (right) Diversity 4, 6, and 8 obtained with TOSD–GSSK modulation with mapping by pairwise disjoint set partitioning; Legend: ($*$) Div=4, $N_t = 4$, $N_a = 2$, $R = 1$ ($\circ$) Div=6, $N_t = 6$, $N_a = 3$, $R = 1$ ($\diamond$) Div=8, $N_t = 5$, $N_a = 4$, $R = 1$.

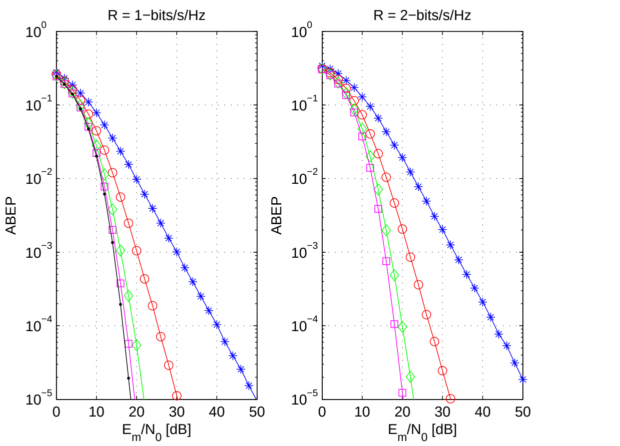

Fig. 2. ABEP against E_m/N_0: performance comparison for the same rate. (left) $R = 1$; Legend: ($*$) Div=1, $N_t = 2$, $N_a = 1$ (SSK), ($\circ$) Div=2, $N_t = 2$, $N_a = 1$ (TOSD–SSK), ($\diamond$) Div=4, $N_t = 4$, $N_a = 2$ (TOSD–GSSK with set partitioning), ($\square$) Div=6, $N_t = 6$, $N_a = 3$ (TOSD–GSSK with set partitioning), ($\cdot$) Div=8, $N_t = 8$, $N_a = 4$ (TOSD–GSSK with set partitioning). (right) $R = 2$; Legend: ($*$) Div=1, $N_t = 4$, $N_a = 1$ (SSK), ($\circ$) Div=2, $N_t = 4$, $N_a = 1$ (TOSD–SSK), ($\diamond$) Div=4, $N_t = 8$, $N_a = 2$ (TOSD–GSSK with set partitioning), ($\square$) Div=6, $N_t = 12$, $N_a = 3$ (TOSD–GSSK with set partitioning).

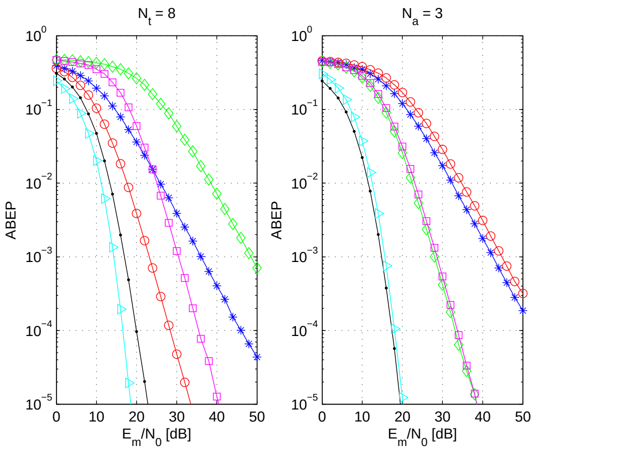

Fig. 3. ABEP against E_m/N_0: performance for the same total and active transmit–antenna. (left) $N_t = 8$; Legend: ($*$) Div=1, $N_a = 1$, $R = 3$ (SSK), ($\circ$) Div=2, $N_a = 1$, $R = 3$ (TOSD–SSK), ($\diamond$) Div=1, $N_a = 4$, $R = 6$ (GSSK), ($\square$) Div=2, $N_a = 4$, $R = 3$ (TOSD–GSSK), ($\cdot$) Div=4, $N_a = 2$, $R = 2$ (TOSD–GSSK with set partitioning), ($>$) Div=8, $N_a = 4$, $R = 1$ (TOSD–GSSK with set partitioning). (right) $N_a = 3$; Legend: ($*$) Div=1, $N_t = 6$, $R = 4$ (GSSK), ($\circ$) Div=1, $N_t = 7$, $R = 5$ (GSSK), ($\diamond$) Div=2, $N_t = 6$, $R = 4$ (TOSD–GSSK), ($\square$) Div=2, $N_t = 7$, $R = 5$ (TOSD–GSSK), ($\cdot$) Div=6, $N_t = 6$, $R = 1$ (TOSD–GSSK with set partitioning), ($>$) Div=6, $N_t = 12$, $R = 2$ (TOSD–GSSK with set partitioning).

In Fig. 3, we compare the performance of various system setups for the same N_t and N_a. We believe that this is an important point to be addressed since some constraints might be imposed on the total and active number of antennas at the transmitter. The results show that, for the same hardware constraints, we can support a broad range of quality–of–service requirements and data rates. This highlights that SSK modulation is a flexible scheme, which can be easily implemented in an adaptive multiple–antenna system design, where we could switch among the SSK, GSSK, TOSD–SSK, TOSD–GSSK, and the more general TOSD–GSSK mapping by pairwise disjoint set partitioning schemes to find the best trade-off among complexity, performance, and data rate.

VIII. Conclusion

In this paper, we have studied the transmit–diversity/multiplexing trade–off of SSK modulation. We have proposed a very flexible modulation scheme, which can accommodate a broad range of data rates, transmit–diversity, and performance requirements. It has been shown that the proposed system achieves a transmit–diversity that is twice the number of active antennas at the transmitter, and a data rate that increases logaritmically with the ratio of total and active antennas at the transmitter. Higher transmission rates can be achieved for systems with transmit–diversity one and two. We believe that the proposed modulation schemes are suitable for an adaptive multiple–antenna wireless system design, where the best quadruple $(N_t, N_a, \mathrm{R}, \mathrm{Div})$ can be tuned according to the specific needs of the end–user.

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